Semicomputable Geometry

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Résumé

Computability and semicomputability of compact subsets of the Euclidean spaces are important notions, that have been investigated for many classes of sets including fractals (Julia sets, Mandelbrot set) and objects with geometrical or topological constraints (embedding of a sphere). In this paper we investigate one of the simplest classes, namely the filled triangles in the plane. We study the properties of the parameters of semicomputable triangles, such as the coordinates of their vertices. This problem is surprisingly rich. We introduce and develop a notion of semicomputability of points of the plane which is a generalization in dimension 2 of the left-c.e. and right-c.e. numbers. We relate this notion to Solovay reducibility. We show that semicomputable triangles admit no finite parametrization, for some notion of parametrization.